



EXPRESSIONS AND EQUATIONS 7 TH GRADE

Lesson 1: Generating Equivalent Expressions

Example 1: Any Order, Any Grouping Property with Addition

- a. Rewrite $5x + 3x$ and $5x - 3x$ by combining like terms.

Write the original expressions and expand each term using addition. What are the new expressions equivalent to?

- b. Find the sum of $2x + 1$ and $5x$.

- c. Find the sum of $-3a + 2$ and $5a - 3$.

Example 2: Any Order, Any Grouping with Multiplication

Find the product of $2x$ and 3 .

Example 3: Any Order, Any Grouping in Expressions with Addition and Multiplication

Use any order, any grouping to write equivalent expressions.

a. $3(2x)$

b. $4y(5)$

c. $4 \cdot 2 \cdot z$

d. $3(2x) + 4y(5)$

e. $3(2x) + 4y(5) + 4 \cdot 2 \cdot z$

- f. Alexander says that $3x + 4y$ is equivalent to $(3)(4) + xy$ because of any order, any grouping. Is he correct? Why or why not?

Lesson 2: Generating Equivalent Expressions

Classwork

Opening Exercise

Additive inverses have a sum of zero. Fill in the center column of the table with the opposite of the given number or expression, then show the proof that they are opposites. The first row is completed for you.

Expression	Opposite	Proof of Opposites
1	-1	$1 + (-1) = 0$
3		
-7		
$-\frac{1}{2}$		
x		
$3x$		
$x + 3$		
$3x - 7$		

Example 1: Subtracting Expressions

- a. Subtract: $(40 + 9) - (30 + 2)$.
- b. Subtract: $(3x + 5y - 4) - (4x + 11)$.

Example 2: Combining Expressions Vertically

- a. Find the sum by aligning the expressions vertically.
 $(5a + 3b - 6c) + (2a - 4b + 13c)$
- b. Find the difference by aligning the expressions vertically.
 $(2x + 3y - 4) - (5x + 2)$

Example 3: Using Expressions to Solve Problems

A stick is x meters long. A string is 4 times as long as the stick.

- a. Express the length of the string in terms of x .

- b. If the total length of the string and the stick is 15 meters long, how long is the string?

Example 4: Expressions from Word Problems

It costs Margo a processing fee of \$3 to rent a storage unit, plus \$17 per month to keep her belongings in the unit. Her friend Carissa wants to store a box of her belongings in Margo's storage unit and tells her that she will pay her \$1 toward the processing fee and \$3 for every month that she keeps the box in storage. Write an expression in standard form that represents how much Margo will have to pay for the storage unit if Carissa contributes. Then, determine how much Margo will pay if she uses the storage unit for 6 months.

Example 5: Extending Use of the Inverse to Division

Multiplicative inverses have a product of 1. Find the multiplicative inverses of the terms in the first column. Show that the given number and its multiplicative inverse have a product of 1. Then, use the inverse to write each corresponding expression in standard form. The first row is completed for you.

Given	Multiplicative Inverse	Proof—Show that their product is 1.	Use each inverse to write its corresponding expression below in standard form.
3	$\frac{1}{3}$	$3 \cdot \frac{1}{3}$ $\frac{3}{1} \cdot \frac{1}{3}$ $\frac{3}{3} = 1$	$12 \div 3$ $12 \cdot \frac{1}{3}$ 4
5			$65 \div 5$
-2			$18 \div (-2)$
$-\frac{3}{5}$			$6 \div \left(-\frac{3}{5}\right)$
x			$5x \div x$
$2x$			$12x \div 2x$

Lesson 3: Writing Products as Sums and Sums as Products

Classwork

Opening Exercise

Solve the problem using a tape diagram. A sum of money was shared between George and Brian in a ratio of 3: 4. If the sum of money was \$56.00, how much did George get?

Example 1

Represent $3 + 2$ using a tape diagram.

Represent $x + 2$ using a tape diagram.

Draw a rectangular array for $3(3 + 2)$.

Draw an array for $3(x + 2)$.

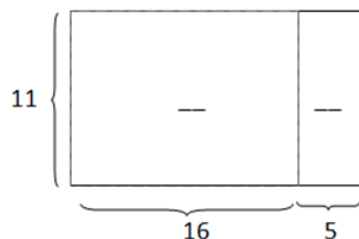
Key Terms

Distributive Property: The distributive property can be written as the identity

$$a(b + c) = ab + ac \text{ for all numbers } a, b, \text{ and } c.$$

Exercise 1

Determine the area of each region using the distributive property.



Example 2

Draw a tape diagram to represent each expression.

a. $(x + y) + (x + y) + (x + y)$

b. $(x + x + x) + (y + y + y)$

c. $3x + 3y$

d. $3(x + y)$

Example 3

Find an equivalent expression by modeling with a rectangular array and applying the distributive property to the expression $5(8x + 3)$.

Exercise 2

For parts (a) and (b), draw an array for each expression and apply the distributive property to expand each expression. Substitute the given numerical values to demonstrate equivalency.

a. $2(x + 1), x = 5$

b. $10(2c + 5), c = 1$

For parts (c) and (d), apply the distributive property. Substitute the given numerical values to demonstrate equivalency.

c. $3(4f - 1), f = 2$

d. $9(-3r - 11), r = 10$

Example 4

Rewrite the expression $(6x + 15) \div 3$ in standard form using the distributive property.

Exercise 3

Rewrite the expressions as a sum.

a. $(2b + 12) \div 2$

b. $(20r - 8) \div 4$

c. $(49g - 7) \div 7$

Example 5

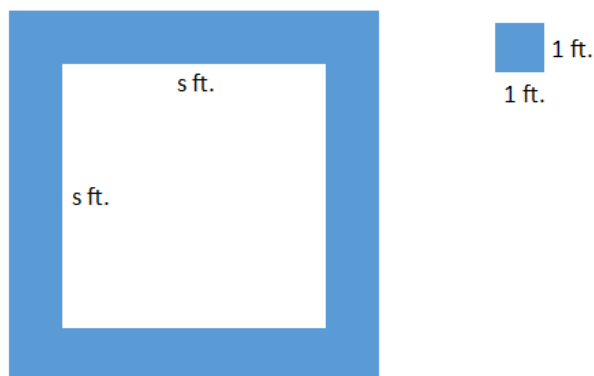
Expand the expression $4(x + y + z)$.

Exercise 4

Expand the expression from a product to a sum by removing grouping symbols using an area model and the repeated use of distributive property: $3(x + 2y + 5z)$.

Example 6

A square fountain area with side length s ft. is bordered by a single row of square tiles as shown. Express the total number of tiles needed in terms of s three different ways.



Lesson 4: Writing Products as Sums and Sums as Products

Classwork

Example 1

a. $2(x + 5)$	
b. $3(x + 4)$	
c. $6(x + 1)$	
d. $7(x - 3)$	
e.	$5x + 30$
f.	$8x + 8$
g.	$3x - 12$
h.	$15x + 20$

Exercise 1

Rewrite the expressions as a product of two factors.

a. $72t + 8$

c. $36z + 72$

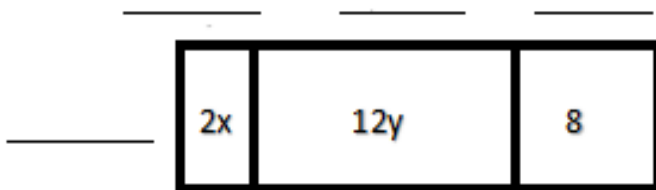
e. $3r + 3s$

b. $55a + 11$

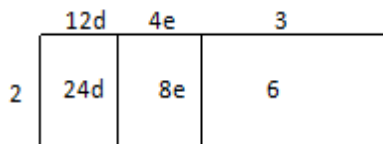
d. $144q - 15$

Example 2

Let the variables x and y stand for positive integers, and let $2x$, $12y$, and 8 represent the area of three regions in the array. Determine the length and width of each rectangle if the width is the same for each rectangle.

**Exercise 2**

- a. Write the product and sum of the expressions being represented in the rectangular array.



- b. Factor $48j + 60k + 24$ by finding the greatest common factor of the terms.

Exercise 3

For each expression, write each sum as a product of two factors. Emphasize the importance of the distributive property. Use various equivalent expressions to justify equivalency.

a. $2 \cdot 3 + 5 \cdot 3$

b. $(2 + 5) + (2 + 5) + (2 + 5)$

c. $2 \cdot 2 + (5 + 2) + (5 \cdot 2)$

d. $x \cdot 3 + 5 \cdot 3$

e. $(x + 5) + (x + 5) + (x + 5)$

f. $2x + (5 + x) + 5 \cdot 2$

g. $x \cdot 3 + y \cdot 3$

h. $(x + y) + (x + y) + (x + y)$

i. $2x + (y + x) + 2y$

Example 3

A new miniature golf and arcade opened up in town. For convenient ordering, a play package is available to purchase. It includes two rounds of golf and 20 arcade tokens, plus \$3.00 off the regular price. There is a group of six friends purchasing this package. Let g represent the cost of a round of golf, and let t represent the cost of a token. Write two different expressions that represent the total amount this group spent. Explain how each expression describes the situation in a different way.

Exercise 4

- a. What is the opposite of $(-6v + 1)$?
- b. Using the distributive property, write an equivalent expression for part (a).

Example 5

Rewrite $5a - (a - 3b)$ in standard form. Justify each step, applying the rules for subtracting and the distributive property.

Exercise 5

Expand each expression and collect like terms.

a. $-3(2p - 3q)$

b. $-a - (a - b)$

Lesson 22 (from Module 2): Solving Equations Using Algebra

Classwork

In this lesson, you will transition from solving equations using tape diagrams to solving equations algebraically by *making zero* (using the additive inverse) and *making one* (using the multiplicative inverse). Justify your work by identifying which algebraic property you used for each step in solving the problems. Explain your work by writing out how you solved the equations step by step and relate each step to those used with a tape diagram.

Example 1: Yoshiro's New Puppy

Yoshiro has a new puppy. She decides to create an enclosure for her puppy in her back yard. The enclosure is in the shape of a hexagon (six-sided polygon) with one pair of opposite sides running the same distance along the length of two parallel flowerbeds. There are two boundaries at one end of the flowerbeds that are 10 ft. and 12 ft., respectively, and at the other end, the two boundaries are 15 ft. and 20 ft., respectively. If the perimeter of the enclosure is 137 ft., what is the length of each side that runs along the flowerbed?

Example 2: Swim Practice

Jenny is on the local swim team for the summer and has swim practice four days per week. The schedule is the same each day. The team swims in the morning and then again for 2 hours in the evening. If she swims 12 hours per week, how long does she swim each morning?

Exercises

Solve each equation algebraically showing each step.

1. $5x + 4 = 19$

2. $15x + 14 = 19$

3. Claire's mom found a very good price on a large computer monitor. She paid \$325 for a monitor that was only \$65 more than half the original price. What was the original price?

4. $2(x + 4) = 18$

5. Ben's family left for vacation after his Dad came home from work on Friday. The entire trip was 600 mi. Dad was very tired after working a long day and decided to stop and spend the night in a hotel after 4 hours of driving. The next morning, Dad drove the remainder of the trip. If the average speed of the car was 60 miles per hour, what was the remaining time left to drive on the second part of the trip?
Remember: Distance = rate multiplied by time.

Lesson 23 (from Module 2): Solving Equations Using Algebra

Classwork

Exercises

1. Youth Group Trip

The youth group is going on a trip to an amusement park in another part of the state. The trip costs each group member \$150, which includes \$85 for the hotel and two one-day combination entrance and meal plan passes.

- Write an equation representing the cost of the trip. Let P be the cost of the park pass.
- Solve the equation algebraically to find the cost of the park pass. Show each step as you solve the equation.
- Model the problem using a tape diagram to check your work.

Suppose you want to buy your favorite ice cream bar while at the amusement park and it costs \$2.89. If you purchase the ice cream bar and 3 bottles of water, and pay with a \$10 bill and receive no change, then how much did each bottle of water cost?

- d. Write an equation to model this situation.
- e. Solve the equation to determine the cost of one water bottle. Show each step as you solve the equation.
- f. Model the problem using a tape diagram to check your work.

2. Weekly Allowance

Charlotte receives a weekly allowance from her parents. She spent half of this week's allowance at the movies, but earned an additional \$4 for performing extra chores. If she did not spend any additional money and finished the week with \$12, what is Charlotte's weekly allowance?

- a. Write an equation that can be used to find the original amount of Charlotte's weekly allowance. Let A be the value of Charlotte's original weekly allowance.
- b. Solve the equation to find the original amount of allowance. Show each step as you solve the equation.
- c. Explain your answer in the context of this problem.
- d. Charlotte's goal is to save \$100 for her beach trip at the end of the summer. Use the amount of weekly allowance you found in part (c) to write an equation to determine the number of weeks that Charlotte must work to meet her goal. Let w represent the number of weeks.
- e. In looking at your answer to part (d), and based on the story above, do you think it will take Charlotte that many weeks to meet her goal? Why or Why not?

3. Travel Baseball Team

Allen is very excited about joining a travel baseball team for the fall season. He wants to determine how much money he should save to pay for the expenses related to this new team. Players are required to pay for uniforms, travel expenses, and meals.

- a. If Allen buys 4 uniform shirts at one time, he gets a \$10.00 discount so that the total cost of 4 shirts would be \$44. Write an algebraic equation that represents the regular price of one shirt. Solve the equation. Show each step as you solve the equation.

- b. What is the cost of one shirt without the discount?

- c. What is the cost of one shirt with the discount?

- d. How much more do you pay per shirt if you buy them one at a time (rather than in bulk)?

Allen's team was also required to buy two pairs of uniform pants and two baseball caps, which total \$68. A pair of pants costs \$12 more than a baseball cap.

- e. Write an equation that models this situation. Let c represent the cost of a baseball cap.

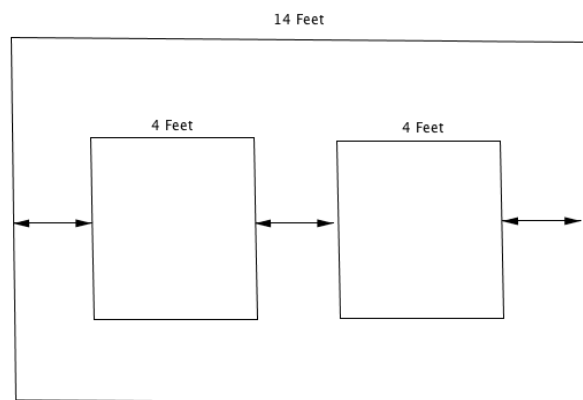
- f. Solve the equation algebraically to find the cost of a baseball cap. Show each step as you solve the equation.
- g. Model the problem using a tape diagram in order to check your work.
- h. What is the cost of one cap?
- i. What is the cost of one pair of pants?

Lesson 7: Understanding Equations

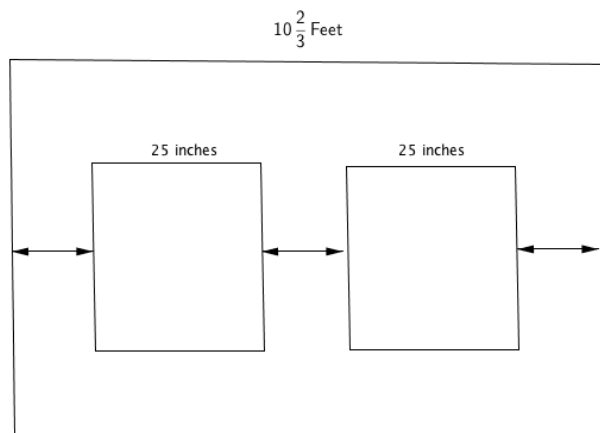
Classwork

Opening Exercise

Your brother is going to college, so you no longer have to share a bedroom. You decide to redecorate a wall by hanging two new posters on the wall. The wall is 14 feet wide and each poster is four feet wide. You want to place the posters on the wall so that the distance from the edge of each poster to the nearest edge of the wall is the same as the distance between the posters, as shown in the diagram below. Determine that distance.



Your parents are redecorating the dining room and want to place two rectangular wall sconce lights that are 25 inches wide along a $10\frac{2}{3}$ feet wall so that the distance between the lights and the distances from each light to the nearest edge of the wall are all the same. Design the wall and determine the distance.



Let the distance between a light and the nearest edge of a wall be x ft. Write an expression in terms of x for the total length of the wall. Then, use the expression and the length of the wall given in the problem to write an equation that can be used to find that distance.

Now write an equation where y stands for the number of *inches*: Let the distance between a light and the nearest edge of a wall be y inches. Write an expression in terms of y for the total length of the wall. Then, use the expression and the length of the wall to write an equation that can be used to find that distance (in inches).

What value(s) of y makes the second equation true: 24, 25, or 26?

Exercise

Sophia pays a \$19.99 membership fee for an online music store.

- a. If she also buys two songs from a new album at a price of \$0.99 each, what is the total cost?

- b. If Sophia purchases n songs for \$0.99 each, write an expression for the total cost.

- c. Sophia's friend has saved \$118 but is not sure how many songs she can afford if she buys the membership and some songs. Use the expression in part (b) to write an equation that can be used to determine how many songs Sophia's friend can buy.

- d. Using the equation written in part (c), can Sophia's friend buy 101, 100, or 99 songs?

Lesson 8: Using If-Then Moves in Solving Equations

Example 1

Julia, Keller, and Israel are volunteer firefighters. On Saturday, the volunteer fire department held its annual coin drop fundraiser at a streetlight. After one hour, Keller had collected \$42.50 more than Julia, and Israel had collected \$15 less than Keller. The three firefighters collected \$125.95 in total. How much did each person collect?

Find the solution using a tape diagram.

What were the operations we used to get our answer?

The amount of money Julia collected is j dollars. Write an expression to represent the amount of money Keller collected in dollars.

Using the expressions for Julia and Keller, write an expression to represent the amount of money Israel collected in dollars.

Using the expressions written above, write an equation in terms of j that can be used to find the amount each person collected.

Solve the equation written above to determine the amount of money each person collected.

Example 2

You are designing a rectangular pet pen for your new baby puppy. You have 30 feet of fence barrier. You decide that you would like the length to be $6\frac{1}{3}$ feet longer than the width.

Draw and label a diagram to represent the pet pen. Write expressions to represent the width and length of the pet pen.

Find the dimensions of the pet pen.

Example 3

Nancy's morning routine involves getting dressed, eating breakfast, making her bed, and driving to work. Nancy spends $\frac{1}{3}$ of the total time in the morning getting dressed, 10 minutes eating breakfast, 5 minutes making her bed and the remaining time driving to work. If Nancy spent $35\frac{1}{2}$ minutes getting dressed, eating breakfast, and making her bed, how long was her drive to work?

Write and solve this problem using an equation.

Is your answer reasonable? Explain.

Example 4

The total number of participants who went on the seventh-grade field trip to the Natural Science Museum consisted of all of the seventh-grade students and 7 adult chaperones. Two-thirds of the total participants rode a large bus and the rest rode a smaller bus. If 54 of them rode the large bus, how many students went on the field trip?

Lesson 9: Using If-Then Moves in Solving Equations

Classwork

Opening Exercise

Heather practices soccer and piano. Each day she practices piano for 2 hours. After 5 days, she practiced both piano and soccer for a total of 20 hours. Assuming that she practiced soccer the same amount of time each day, how many hours per day, h , did Heather practice soccer?

Over 5 days, Jake practices piano for a total of 2 hours. Jake practices soccer for the same amount of time each day. If he practiced piano and soccer for a total of 20 hours, how many hours, h , per day did Jake practice soccer?

Example 1

Fred and Sam are a team in the local 138.2 mile bike-run-athon. Fred will compete in the bike race, and Sam will compete in the run. Fred bikes at an average speed of 8 miles per hour and Sam runs at an average speed of 4 miles per hour. The bike race begins at 6:00 a.m., followed by the run. Sam predicts he will finish the run at 2:33 a.m. the next morning.

- How many hours will it take them to complete the entire bike-run-athon?
- If t is how long it takes Fred to complete the bike race, in hours, write an expression to find Fred's total distance.
- Write an expression, in terms of t to express Sam's time.

- d. Write an expression, in terms of t , that represents Sam's total distance.
- e. Write and solve an equation using the total distance both Fred and Sam will travel.
- f. How far will Fred bike, and how much time will it take him to complete his leg of the race?
- g. How far will Sam run and how much time will it take him to complete his leg of the race?

Total Time (hours)	Fred's Time (hours)	Sam's Time (hours)
10	6	
15	12	
20	8	
18.35	8	
20.55	t	

Example 2

Shelby is seven times as old as Bonnie. If in 5 years, the sum of Bonnie and Shelby's ages is 98, find Bonnie's present age. Use an algebraic approach.

Lesson 12: Properties of Inequalities

Classwork

Example 1

Preserves the inequality symbol:

Reverses the inequality symbol:

Station 1

Die 1	Inequality	Die 2	Operation	New Inequality	Inequality Symbol Preserved or Reversed?
−3	<	5	Add 2	$-3 + 2 < 5 + 2$ $-1 < 7$	Preserved
			Add −3		
			Subtract 2		
			Subtract −1		
			Add 1		

Examine the results. Make a statement about what you notice, and justify it with evidence.

Station 2

Die 1	Inequality	Die 2	Operation	New Inequality	Inequality Symbol Preserved or Reversed?
-3	<	4	Multiply by -1	$(-1)(-3) < (-1)(4)$ $3 < -4$ $3 > -4$	Reversed
			Multiply by -1		
			Multiply by -1		
			Multiply by -1		
			Multiply by -1		

Examine the results. Make a statement about what you notice and justify it with evidence.

Station 3

Die 1	Inequality	Die 2	Operation	New Inequality	Inequality Symbol Preserved or Reversed?
-2	>	-4	Multiply by $\frac{1}{2}$	$(-2)\left(\frac{1}{2}\right) > (-4)\left(\frac{1}{2}\right)$ $-1 > -2$	Preserved
			Multiply by 2		
			Divide by 2		
			Divide by $\frac{1}{2}$		
			Multiply by 3		

Examine the results. Make a statement about what you notice, and justify it with evidence.

Station 4

Die 1	Inequality	Die 2	Operation	New Inequality	Inequality Symbol Preserved or Reversed?
3	$>$	-2	Multiply by -2	$3(-2) > (-2)(-2)$ $-6 < 4$	Reversed
			Multiply by -3		
			Divide by -2		
			Divide by $-\frac{1}{2}$		
			Multiply by $-\frac{1}{2}$		

Examine the results. Make a statement about what you notice and justify it with evidence.

Exercise

Complete the following chart using the given inequality, and determine an operation in which the inequality symbol is preserved and an operation in which the inequality symbol is reversed. Explain why this occurs.

Inequality	Operation and New Inequality Which Preserves the Inequality Symbol	Operation and New Inequality Which Reverses the Inequality Symbol	Explanation
$2 < 5$			
$-4 > -6$			
$-1 \leq 2$			
$-2 + (-3) < -3 - 1$			

Lesson 13: Inequalities

Classwork

Opening Exercise: Writing Inequality Statements

Tarik is trying to save \$265.49 to buy a new tablet. Right now, he has \$40 and can save \$38 a week from his allowance.

Write and evaluate an expression to represent the amount of money saved after ...

2 weeks

3 weeks

4 weeks

5 weeks

6 weeks

7 weeks

8 weeks

When will Tarik have enough money to buy the tablet?

Write an inequality that will generalize the problem.

Example 1: Evaluating Inequalities—Finding a Solution

The sum of two consecutive odd integers is more than -12 . Write several true numerical inequality expressions.

The sum of two consecutive odd integers is more than -12 . What is the smallest value that will make this true?

- a. Write an inequality that can be used to find the smallest value that will make the statement true.

- b. Solve the inequality written in part (a).

Exercises

1. Connor went to the county fair with a \$22.50 in his pocket. He bought a hot dog and drink for \$3.75, and then wanted to spend the rest of his money on ride tickets, which cost \$1.25 each.
 - a. Write an inequality to represent the total spent where r is the number of tickets purchased.

 - b. Connor wants to use this inequality to determine whether he can purchase 10 tickets. Use substitution to show whether he will have enough money.

 - c. What is the total maximum number of tickets he can buy based upon the given information?

2. Write and solve an inequality statement to represent the following problem:
On a particular airline, checked bags can weigh no more than 50 pounds. Sally packed 32 pounds of clothes and five identical gifts in a suitcase that weigh 8 pounds. Write an inequality to represent this situation.

Lesson 14: Solving Inequalities

Classwork

Opening Exercise

The annual County Carnival is being held this summer and will last $5\frac{1}{2}$ days. Use this information and the other given information to answer each problem.

You are the owner of the biggest and newest rollercoaster called the Gentle Giant. The rollercoaster costs \$6 to ride. The operator of the ride must pay \$200 per day for the ride rental and \$65 per day for a safety inspection. If you want to make a profit of at least \$1,000 each day, what is the minimum number of people that must ride the rollercoaster?

Write an inequality that can be used to find the minimum number of people, p , that must ride the rollercoaster each day to make the daily profit.

Solve the inequality.

Interpret the solution.

Example 1

A youth summer camp has budgeted \$2,000 for the campers to attend the carnival. The cost for each camper is \$17.95, which includes general admission to the carnival and two meals. The youth summer camp must also pay \$250 for the chaperones to attend the carnival and \$350 for transportation to and from the carnival. What is the greatest number of campers who can attend the carnival if the camp must stay within its budgeted amount?

Example 2

The carnival owner pays the owner of an exotic animal exhibit \$650 for the entire time the exhibit is displayed. The owner of the exhibit has no other expenses except for a daily insurance cost. If the owner of the animal exhibit wants to make more than \$500 in profits for the $5\frac{1}{2}$ days, what is the greatest daily insurance rate he can afford to pay?

Example 3

Several vendors at the carnival sell products and advertise their businesses. Shane works for a recreational company that sells ATVs, dirt bikes, snowmobiles, and motorcycles. His boss paid him \$500 for working all of the days at the carnival plus 5% commission on all of the sales made at the carnival. What was the minimum amount of sales Shane needed to make if he earned more than \$1,500?

Lesson 15: Graphing Solutions to Inequalities

Classwork

Exercise 1

Two identical cars need to fit into a small garage. The opening is 23 feet 6 inches wide, and there must be at least 3 feet 6 inches of clearance between the cars and between the edges of the garage. How wide can the cars be?

Example

A local car dealership is trying to sell all of the cars that are on the lot. Currently, there are 525 cars on the lot, and the general manager estimates that they will consistently sell 50 cars per week. Estimate how many weeks it will take for the number of cars on the lot to be less than 75.

Write an inequality that can be used to find the number of full weeks, w , that it will take for the number of cars to be less than 75. Since w is the number of full or complete weeks, $w = 1$ means at the end of week 1.

Solve and graph the inequality.

Interpret the solution in the context of the problem.

Verify the solution.

Exercise 2

The cost of renting a car is \$25 per day plus a one-time fee of \$75.50 for insurance. How many days can the car be rented if the total cost is to be no more than \$525?

- a. Write an inequality to model the situation.

- b. Solve and graph the inequality.

- c. Interpret the solution in the context of the problem.

Additional Exercises

For each problem, write, solve, and graph the inequality, and interpret the solution within the context of the problem.

1. Mrs. Smith decides to buy three sweaters and a pair of jeans. She has \$120 in her wallet. If the price of the jeans is \$35, what is the highest possible price of a sweater, if each sweater is the same price?

2. The members of the Select Chorus agree to buy at least 250 tickets for an outside concert. They buy 80 less lawn tickets than balcony tickets. What is the least number of balcony tickets bought?
3. Samuel needs \$29 to download some songs and movies on his iPod. His mother agrees to pay him \$6 an hour for raking leaves in addition to his \$5 weekly allowance. What is the minimum number of hours Samuel must work in one week to have enough money to purchase the songs and movies?